NEW JERSEY CENTER
FOR TEACHING & LEARNING

Progressive Mathematics Initiative® (PMI®)

MATH6447: Learning and Teaching Precalculus

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Course Credit: 3.0 NJCTL credits

Dates & Times:
This is a 3-credit, self-paced course, covering 7 modules of content. The exact number of hours that you can expect to spend on each module will vary based upon the module coursework, as well as your study style and preferences. You should plan to spend approximately 15 hours per credit working online, and up to 30 hours per credit working offline.

Graduate Student Handbook: www.njctl.org/graduate-handbook/

COURSE DESCRIPTION:

This course is for teachers to learn the content of PMI Precalculus and how to teach that course to students. Topics include functions; polynomial & rational functions; exponential & logarithmic functions; systems & matrices; vectors; sequences & series; and probability & statistics.

STUDENT LEARNING OUTCOMES:

Upon completion of the course, the student will be able to:

1. Apply the principles of mathematics in the areas of graphical analysis, graphing functions/equations, solving equations and writing algebraic proofs, as outlined in the module learning outcomes below.
2. Apply student-centered pedagogy to teach mathematics to students.
3. Apply mathematical tools commonly used in precalculus including arithmetic operations, inverse operations, algebraic proofs, problem solving and function/equation analysis.
4. Identify, understand, and communicate the elements, representations, and models of equations and functions to solve word problems.
5. Examine, investigate, and assess the relationships between various mathematical models and their variables.

TEXTS, READINGS, INSTRUCTIONAL RESOURCES

Required Texts:

- This course uses a free digital textbook accessible at: https://njctl.org/courses/math/pre-calculus/

Recommended Readings:

- Related articles within short answer assignments.

COURSE REQUIREMENTS:

In order to receive a Passing grade, the participant must complete the following course requirements:

1. Activities: A number of different learning activities will ensure participant engagement and learning in the course. These include:
   - Engage in video module lessons which demonstrate minimized direct instruction followed by frequent formative assessment.
   - Completion of formative assessments aligned to learning objectives which include detailed analysis when answered incorrectly.
   - Interaction with module discussion boards that allow conversation with peers and course instructors about the module's content, delivering that content to students. Discussion boards also serve as a place to ask and answer questions related to the module's content.
2. Short Answer Assignment: Each module requires one (1) original response to a given prompt. These prompts are typically based upon course lessons and require teachers to analyze, reflect, and make connections between the module's content and their own classroom practice.
3. Mastery Exercises: For each module, these multiple-choice question quizzes assess the content knowledge gained in a module. Participants have the opportunity to retake; random questions are pulled from a larger question bank on each attempt ensuring varied questions.
4. Virtual Labs: In each module, a virtual lab write-up will be submitted. Virtual labs are interactive lab simulations that promote discovery-based student learning through real-world applications and analysis.
5. Module Exam: One is completed at the end of each module. It is a culminating exam consisting of multiple choice and free response questions aligned to the standards and objectives of the module.
6. Reflection Paper: At the end of the course, participants are required to reflect on the knowledge taught in the course, make connections, and compare/contrast their current pedagogy with new strategies gained in this assignment.
7. Final Exam: At the end of the course, a comprehensive exam consisting of Multiple Choice and Free Response questions assesses the content knowledge learned throughout the course.

GRADE DISTRIBUTION AND SCALE:

In order to receive a Passing grade, the participant must complete the following course requirements: all short answer assignments, mastery exercises, labs, exams, and the reflection paper outlined in the *Assignments* section of the Class Schedule (below).

Grade Distribution:

Module Exams	70%
Final Exam	10%

Labs	6%
Short Answer Assignments	6%
Mastery Exercises	6%
Reflection Paper	2%

Grade Scale:

A	93 – 100
A-	90 – 92
B+	86 – 89
B	83 – 86
B-	80 – 82
C+	77 – 79
C	73 – 76
C-	70 – 72
D	60.0 – 69.9
F	59.9 or below

GRADING RUBRIC:
The following rubric is used to score:

- Short Answer Assignment – 6% of grade
- Reflection Paper – 2% of grade

The minimum possible score for this rubric is 4 points, and the score will be converted to the minimum grade available in this module (which is zero unless the scale is used). The maximum score of 25 points will be converted to the maximum grade.

Intermediate scores will be converted respectively and rounded to the nearest available grade. If a scale is used instead of a grade, the score will be converted to the scale elements as if they were consecutive integers.

	Meets Expectation	**Approaches Expectation**	**Below Expectation**	**Limited Evidence**
	7 points	***5 points***	***3 points***	***1 point***
Content	• Demonstrates excellent knowledge of concepts, skills, and theories relevant to topic.	• Demonstrates fair knowledge of concepts, skills, and theories.	• Demonstrates incomplete or insubstantial knowledge of concepts, skills, and theories.	• Demonstrates little or no knowledge of concepts, skills, and theories.
Depth of Reflection	• Content is well supported and addresses all required components of the assignment.	• Content is partially supported; addresses most of the required components of the assignment.	• Content contains major deficiencies; addresses some of the required components of the assignment.	• Content is not supported and/or includes few of the required components of the assignment.
Evidence and Practice	• Response shows strong evidence of synthesis of ideas presented and insights gained throughout the entire course. The implications of these insights for the respondent's overall teaching practice are thoroughly detailed, as applicable.	• Writing is mostly clear, concise, and well organized with good sentence/paragraph construction. Thoughts are expressed in a coherent and logical manner. There are no more than five spelling, grammar, or syntax errors per page of writing.	• Response is missing some components and/or does not fully meet the requirements indicated in the instructions. Some questions or parts of the assignment are not addressed. Some attachments and additional documents, if required, are missing or unsuitable for the purpose of the assignment.	• Response excludes essential components and/or does not address the requirements indicated in the instructions. Many parts of the assignment are addressed minimally, inadequately, and/or not at all.

	4 points	*3 points*	*2 points*	*1 point*
Writing Quality	• Writing is well-organized, clear, concise, and focused; no errors.	• Some minor errors or omissions in writing organization, focus, and clarity.	• Some significant errors or omissions in writing organization, focus, and clarity.	• Numerous errors in writing organization, focus, and/or clarity.

The following rubric is used to score:

- Labs – 6% of grade

The minimum possible score for this rubric is 2 points, and the score will be converted to the minimum grade available in this module (which is zero unless the scale is used). The maximum score of 14 points will be converted to the maximum grade.

Intermediate scores will be converted respectively and rounded to the nearest available grade. If a scale is used instead of a grade, the score will be converted to the scale elements as if they were consecutive integers.

	Meets Expectation	**Approaches Expectation**	**Below Expectation**	**Limited Evidence**
	7 points	***5 points***	***3 points***	***1 point***
Completeness	• Lab write-up is complete with no missing fields.	• Lab write-up has 1-2 missing fields.	• Lab write-up has 3-5 missing fields.	• There are more than 5 missing fields on the lab write-up.
Calculations	• All answers are calculated correctly.	• Most answers are calculated correctly, but there are 1-2 minor calculation errors.	• Most answers are calculated correctly, but there are multiple minor calculation errors, or 1-2 gross miscalculations.	• There are calculation errors throughout the lab.

The remaining types of assignments are not scored using a rubric. These assignments are scored using percentage correct to assign a letter grade. The assignments in this manner are as follows:

- Mastery Exercises – 6% of grade
- Module Exams – 70% of grade
- Final Exam – 10% of grade

Mastery Exercises can be retaken as many times as desired to ensure a high score. Due to the nature of these assignments, each time they are taken, they will be composed of ten unique questions pulled randomly from a larger question bank.

Module and Final Exams are scored using a curve, which allows us to keep content exams rigorous. Module Exams can be retaken one time. Final Exams cannot be retaken.

ACADEMIC STANDING:
NJCTL has established standards for academic good standing within a student's academic program. Students enrolled in any NJCTL online course must receive an 80 or higher to successfully complete a course and receive credit for that course. An 80 is equivalent to a GPA of 2.7 or B-. Additionally, students in an endorsement program must receive a cumulative GPA of 3.0 for all courses combined in order to successfully complete the program.

ACADEMIC INTEGRITY:
Students must assume responsibility for maintaining honesty in all work submitted for credit and in any other work designated by the instructor of the course. Academic dishonesty includes cheating, fabrication, facilitating academic dishonesty, plagiarism, reusing /repurposing your own work, unauthorized possession of academic materials, and unauthorized collaboration.

CITING SOURCES WITH APA STYLE:
All students are expected to follow proper writing and APA requirements when citing in APA (based on the APA Style Manual, 6th edition) for all assignments.

DISABILITY SERVICES STATEMENT:
We are committed to providing reasonable accommodations for all persons with disabilities. Any student with a documented disability requesting academic accommodations should contact the Dean of Students, Melissa Axelsson, for additional information to coordinate reasonable accommodations for students with documented disabilities (melissa@njctl.org).

NETIQUETTE:
Respect the diversity of opinions among the instructor and classmates and engage with them in a courteous, respectful, and professional manner. All posts and classroom communication must be conducted in accordance with the student code of conduct. Think before you push the Send button. Did you say just what you meant? How will the person on the other end read the words?

Maintain an environment free of harassment, stalking, threats, abuse, insults or humiliation toward the instructor and classmates. This includes, but is not limited to, demeaning written or oral comments of an ethnic, religious, age, disability, sexist (or sexual orientation), or racist nature; and the unwanted sexual advances or intimidations by email, or on discussion boards and other postings within or connected to the online classroom.

If you have concerns about something that has been said, please let your instructor know.

CLASS SCHEDULE:

Module	Module Learning Outcomes	Assignments
1 – Functions	• Identify functions from points and graphs. • Evaluate a function for a given value. • Identify the domain and range of a given equation or graph. • Identify key components of a piecewise function. • Evaluate piecewise functions for given values. • Identify the equations and graphs of elementary/parent functions. • Solve for the solutions/intersections to given graphs and equations. • Use technology to graph and solve functions. • Identify where functions' graphs are increasing/decreasing. • Identify if a function is odd or even. • Evaluate the sum/difference and the product of odd/even functions. • Evaluate average rate of change. • Modify equations for functions to account for given transformations. • Combine functions using arithmetic operations. • Identify the domain of combined functions. • Evaluate composite functions. • Create composite functions.	• Short Answer • Lab • Mastery Exercise • Module Exam
2 – Polynomial & Rational Functions	• Prove if a function is even or odd. • Identify the zeros/roots of a polynomial function given equations, tables, and graphs. • Identify the maximas/minimas of a polynomial functions' graph. • Identify if a function is bounded/unbounded. • Utilize the Intermediate Value Theorem to identify the zeros of a given function. • Use synthetic substitution to evaluate a standard form polynomial at a given argument. • Use synthetic substitution to determine the zeros of a polynomial function. • Evaluate the rational zeros of a polynomial function using the Rational Zeros Theorem. • Write polynomials from its given zeros. • Apply the Remainder Theorem, the Factor Theorem and Synthetic Division to complex numbers. • Graph polynomial functions and identify key features of the graph; end behavior, increasing/decreasing intervals, and intercepts. • Determine if a rational function has any domain/range restrictions or removable discontinuity. • Identify the domain and range of a rational function. • Evaluate the intercepts of a rational function. • Identify/evaluate the asymptotes of a rational function. • Rewrite rational functions into partial fractions. • Simplify complex fractions. • Solve rational equations and determine if there are any extraneous solutions. • Solve rational inequalities by graphing or using a sign chart.	• Short Answer Assignment • Lab • Mastery Exercise • Module Exam

3 – Exponential & Logarithmic Functions	• Apply properties of exponents to simplify/rewrite expressions. • Solve equations using the properties of exponents. • Evaluate exponential functions for given values. • Write the equation of an exponential function from its graph. • Describe transformations of exponential functions from their graphs and equations. • Identify key features of exponential functions' graphs; horizontal/vertical shifts, asymptote, intercepts, growth/decay, domain/range, etc. • Calculate compound interest. • Utilize the constant e to evaluate continuously compounded interest. • Evaluate expressions that contain the constant e. • Describe transformations of exponential functions that contain the constant e from their graphs and equations. • Identify key features of exponential functions' graphs that contain the constant e ; horizontal/vertical shifts, asymptote, intercepts, growth/decay, domain/range, etc. • Convert between exponential and logarithmic form. • Solve for missing values in logarithmic and exponential functions. • Identify key components of logarithmic functions in algebraic and graph form. • Graph logarithmic functions. • Create a table of values for a logarithmic function. • Describe transformations of logarithmic functions from their graphs and equations. • Apply properties of logarithms to simplify, expand, condense, and solve expressions. • Identify extraneous solutions of logarithmic equations. • Utilize exponential and logarithmic functions to model various situations; growth, decay, compound interest, half-life, etc.	• Short Answer Assignment • Lab • Mastery Exercise • Module Exam
4 – Matrices & Systems	• Describe matrices based on their dimensions. • Utilize matrix notation to describe a matrix and identify specific entries in a matrix. • Evaluate scalar multiplication for any given matrix. • Add/subtract matrices. • Solve for missing values in given matrices. • Perform multiple arithmetic operations on given matrices. • Multiply matrices. • Determine if two matrices can be multiplied. • Evaluate the product of two given matrices. • Evaluate the determinant of 2x2 and 3x3 matrices, including the use of Cramer's rule for 3x3 matrices. • Find the inverse of 2x2 and 3x3 matrices and prove that two matrices are inverses. • Utilize matrix operations and Cramer's rule to solve systems of equations.	• Short Answer Assignment • Lab • Mastery Exercise • Module Exam

5 – Sequences & Series	● Identify finite/infinite sequences. ● Solve for missing terms in an arithmetic sequence. ● Create explicit formulas for arithmetic sequences. ● Apply arithmetic sequences to real-life applications. ● Solve for missing terms in a geometric sequence. ● Create recursive/explicit formulas for geometric sequences. ● Apply geometric sequences to real-life applications. ● Identify special sequences, including Fibonacci Sequences. ● Evaluate the arithmetic/geometric series for a given sequence. ● Solve for missing terms in an arithmetic/geometric series. ● Determine the coefficient of any given term in binomial expansion.	● Short Answer Assignment ● Lab ● Mastery Exercise ● Module Exam
6 – Probability & Statistics	● Summarize, represent, and interpret data collected from measurements on a single variable (e.g., box plots, dot plots, normal distributions). ● Summarize, represent, and interpret data collected from measurements on two variables, either categorical or quantitative (e.g., scatterplots, time series). ● Create and interpret linear regression models (e.g., rate of change, intercepts, correlation coefficient). ● Understand statistical processes and how to evaluate them. ● Make inferences and justify conclusions from samples, experiments, and observational studies. ● Understand the concepts of independence and conditional probability and how to apply these concepts to data. ● Compute probabilities of simple events, probabilities of compound events, and conditional probabilities. ● Make informed decisions using probabilities and expected values. ● Use simulations to construct experimental probability distributions and to make information inferences about theoretical probability distributions. ● Find probabilities involving finite sample spaces and independent trials.	● Short Answer Assignment ● Lab ● Mastery Exercise ● Module Exam
7 – Reflection & Final Exam	● Review course topics ● Zoom call with instructor, as needed	● Reflection Paper ● Final Exam