Imparting Meaning to Mathematics:

Student Perceptions of the Effect of Ninth-Grade Physics on Learning Mathematics

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The United States government has made improved achievement in mathematics and science a national priority. One approach towards achieving that involves coordinating the instruction of science and mathematics. Toward that end, one school is teaching an algebra-intensive physics course to ninth-grade students. The school's expectation is that both science and mathematics will benefit from this approach over the four years of a student's high school education. This study focused on how students currently enrolled in that physics course perceive that it has affected their understanding of mathematics. The data consisted of individual or group interviews with ten students augmented by quantitative data as to their current and prior understanding of mathematics. It was found that all the students perceived that the physics course had benefited them. Importantly, their perceptions regarding that benefit systematically varied according to their prior and current level of achievement in mathematics. This physics course was found, therefore, to represent an opportunity for a wide range of students to improve their mathematics, in varying ways, while working together in a single class organized along social constructivist lines.

The U.S. is joining other nations in making mathematics and science achievement educational priorities, side by side with reading and language arts (Robitaille & Travers, 1992, pg. 117). The first high school assessments, required in 2004/2005 by the "No Child Left Behind Act", are in reading, language arts and mathematics. The next requirement is for science, which is to be assessed by 2007/2008.

A reform effort relating to the science curriculum is underway in many school districts nationwide. Rather than the traditional order of biology – chemistry – physics, these districts are teaching physics in the first year and biology in the third year (Pasero, 2001, pg. 7). As this change gathers momentum, it is being instituted in high schools nationally. The San Diego school district implemented this new science sequence in all of its secondary schools in 2004.

The original thrust was to improve the teaching and learning of science. The relative mathematical sophistication required for ninth-graders to succeed in a physics course, as compared to a biology course, was considered a negative. However, an alternative perspective is that it may be possible to teach the physics course in a way that benefits achievement in mathematics.

This may be determined by the manner in which ninth-grade physics is taught. At the outset, many conceived of freshman physics as requiring a "conceptual" approach. This was to address the concern that few students understand trigonometry by the ninth grade and traditional first year physics classes require trigonometry. An alternative approach is to teach physics using a mathematically rigorous curriculum confined to algebra. Since most students study algebra by ninth grade, teaching physics built on algebra concepts presents an opportunity to improve understanding in both subjects.

Algebra represents the foundation of much of the mathematics that follows. Students who understand the meaning of, and how to work with and generate, algebraic expressions are well on their way to becoming successful in mathematics. This represents an important transition from procedural to structural mathematics for many students. "Procedural refers to arithmetic operations carried out on numbers to yield numbers…. Structural, on the other hand, refers to a different set of operations that are carried out, not on numbers, but on algebraic expressions" (Kieran, 1992, pg. 392).

This study investigates the attempt of one school to use the revised sequence of science courses to benefit its mathematics program. That school has structured its science and mathematics curricula such that all students study physics in ninth grade. More than half of these students qualify for a mathematically rigorous physics course through a combination of test scores and middle school grades. In that physics class, constructivist techniques are used to interconnect four domains: quantitative physics, qualitative physics, procedural algebra and structural algebra. The physics content is considered secondary to the building of conceptual understandings and connections between these four domains.

The purpose of this study is to determine whether and, if so, in what ways students believe that their ninth-grade algebra-based physics course has affected them with regard to mathematics. The answer to this question may contribute to discussions taking place in districts that are considering revising the sequence and curricula of their science courses.

Literature Review

If a student's achievement in a mathematics course is to be affected by their learning in a physics course, transfer must take place between those courses. Therefore, the efficacy of transfer is fundamental to this study. The literature on transfer is discussed below along with three possible pathways by which a mathematically intensive physics course could lead to benefits in mathematics. These three pathways are titled *Practice*, *Usefulness* and *Meaning.*

Transfer

"Transfer cannot be distinguished from learning. Teaching and learning are a tapestry that does not lend itself to such labels as near transfer and far transfer" (Butterfield, Slocum, & Nelson, 1993, pg. 219). If education in general, and science or mathematics education in particular, is to have more applicability and value than teaching students to solve specific textbook problems in a specific classroom on a specific day, transfer is critical.

Gick and Holyoak (1980) found that subjects demonstrated significant transfer between domains through the use of analogical thinking. They later determined that transfer could be further improved by using two analogous training stories, combined with having the subject summarize the principle connecting those two stories (Gick & Holyoak, 1983).

Bassok and Holyoak (1989) found strong transfer from algebra to physics, but almost no transfer from physics to algebra. However, Bassok later recognized a flaw in the design of that study. She redid it and found strong transfer both from algebra to physics and from physics to algebra (Bassok, 1990).

Smith and Unger (1997) found that “asking students to make connections between two domains that are not equivalently understood typically enhances understanding of the less well understood domain in a supportive, socially scaffolded instructional context”.

Ploetzner and VanLehn (1997) measured transfer between conceptual and quantitative understandings of physics and found that “the degree of transfer that occurred between them was 41%. This is comparable to other studies of transfer from standard physics training to qualitative understanding” (Ploetzner & VanLehn, 1997, pg. 175).

Dibble, et al. found significant transfer among adult men when they were motivated to learn and were in an open risk-free environment (Dibble, Glaser, Gott, Hall, & Pokorny, 1993). They also found that “the primary content of transfer takes the form of abstract knowledge representations…. good learners access their existing mental models…. They then used these models to guide their performance as they crafted solutions to new problems” (pg. 286).

Practice

The physics course provides an increased opportunity for students who are skilled in mathematics to practice those skills on a range of problems.

> Given more examples, students can form a frame (or script or schema) or a set of discriminative stimuli that make an appropriate response more likely for subsequent problems. Viewing choice of solution to later problems as a matter of analogical reasoning, we can say that more examples allow structural features

and goals to affect access to a learned solution and to guide appropriate mapping onto later problems (Carbonell, 1986; Holyoak, 1985)" (Butterfield et al., 1993, pg. 219).

Kieran pointed out the challenge of establishing transfer between qualitative and quantitative understandings of mathematics as well as to science, stating that "generating equations to represent the relationships found in typical word problems is well know to be one of the major areas of difficulty for high school algebra students" (1992, pg. 403). She partially attributed this to the failure to make connections between academic classes.

Marrongelle (2004) posed the question, "how [do] students in an integrated calculus and physics class use physics to help them solve calculus problems" (pg. 258). She monitored eight students in an integrated physics / calculus course and found that "some students introduce contexts to solve mathematics problems; this result suggests that students can use contexts in meaningful ways to solve mathematics problems, contrary to past research that has pointed out the difficulty students have solving problems in context" (pg. 258).

Usefulness

Students who doubt the value of mathematics are less likely to be motivated to learn it. However, the coordinated instruction of physics and algebra serves to expose students to applications of algebra concepts as they are taught. This could increase their drive to learn algebra, in particular, and mathematics, in general, by showing its usefulness, an important aspect of motivation. "Some of the most widely decried failures of transfer - failure to apply knowledge learned in school to practical problems encountered in everyday life - may largely reflect that material taught in school is often disconnected from any clear goal (Gick & Holyoak, 1987, pg. 31)" (Butterfield et al., 1993, pg. 232).

While the goal of being able to solve physics problems, is itself confined to the school environment, it reflects a task that requires algebra, and thereby represents a benefit of learning algebra. Further to that point, "…freshman who enjoy physics often see that math will help them understand it better, getting them excited about math as well as science" (Pasero, 2001, pg. 15).

Meaning

Physics and mathematics share the common goal of teaching students to become good “problem-solvers”. Alan Van Heuvelen (1991) calls this teaching students to “learn to think like physicists” (pg. 892). Schoenfeld (1992) described this as “help[ing] students to develop a mathematical point of view… that develop[s] their analytical skills, and the ability to reason in extended chains of argument” (pg. 345). In 1999, Redish said, “physics is really about building mental maps that allow us to make sense of the world. To do this we have to create map structures that match not only what happens in the physical world but the ways we can comfortably think about it” (pg. 570).

Reed, Dempster and Ettinger (1985) studied the use of analogous problems for solving mathematical word problems. Students who understood the reasoning behind the solution of the training problem achieved the best results. Memorizing a procedure did not lead to transferable problem solving strategies. If problem solving is the goal, having students memorize procedures for solving specific problems is ineffective.

Sherin (2001) argued that mathematics and physics are inextricably intertwined.

> “Mathematical expressions are part of the very language of physics” (pg. 480). “Successful students learn to understand what equations say in a fundamental sense; they have a feel for expressions, and this understanding guides their work…. We do students a disservice by treating conceptual understanding as separate from the use of mathematical notations” (pg. 482) and finally “…naïve physics knowledge provides part of the conceptual basis in terms of which equations are understood” (pg. 483).

Methodology

The purpose of this study was to determine student perceptions regarding the effect of taking a mathematically intensive 9^{th} grade physics course on their achievement in mathematics. A site was located where students were enrolled in just such a physics course. Interviews were conducted with ten students who were enrolled in that course. That data was analyzed qualitatively and quantitatively and found to reveal significant patterns.

The Research Site

The setting for this study was a county-run suburban vocational/magnet high school located near a major US city. The school has a population of between 600 and 700 students in grades nine through twelve. Approximately 180 of these students were in ninth grade at the time of the study.   The school attracts students that are interested in a technical or vocational education. They must be a resident of the county and apply to the school. Their acceptance depends on their middle school academic results, their test scores and the program to which they apply. The school runs programs that span a range from highly technical, requiring a strong science and mathematics foundation, to traditional vocational.

All the students in the school can take any of its academic courses, but students in the more technical programs are required to take the very challenging science and mathematics courses. The school has instituted an algebra-based physics course, Physics Honors, which is taken by all qualified students, regardless of their program of study. Just over half of the entering ninth-graders qualified for that course last year by virtue of their middle school grades and an entrance examination in mathematics. Those who did not qualify took a more conceptual, less mathematical, physics course. All the students who took the algebra-based physics course were either taking an intensive algebra course, Honors Algebra, at the same time or took it in the prior year.

The Researcher

The school employs me as the Chair for Science and Engineering and as a physics teacher. In that role, I designed the mathematics and science sequence at the school in a process that began six years ago and continues today. I have had a long-term interest in this topic. Since the school and the school district share my interest in the efficacy of this program, access was not a problem.

The benefit of my role on campus was that it made the logistics of organizing interviews relatively simple. However, I was concerned with the various forms of bias that could be generated by my role. I identified four possible types of bias and did my best to minimize each of them. These were:

- My role as a participant's teacher might make it difficulty for he or she to be open with me and not try to please me with their answers.

- My role in the school might make it difficult for students to be open with me and not try to please me with their answers.
- As the principal architect of the mathematics and science sequence, I might design questions that would steer the responses of the students in a way that I would prefer.
- As the principal architect of the mathematics and science sequence, I might react to student responses in a way that steered their later responses in a way that I would prefer.

The first concern was easily addressed by not using any of my students in this study. While I teach two sections of Honors Physics course, there are four other sections taught by two other teachers. Subjects were recruited from one of those other teacher's two classes so that none of my own students were participants. I had no relationship with the subjects prior to conducting these interviews.

The last three concerns were addressed in two ways. First, by making a conscious effort to minimize the bias in my questions and responses to participants' answers. Second, I transcribed and analyzed each interview before conducting the next one. This gave me a chance to study my questions, and reactions to participant answers, and use that feedback to reduce the bias in my next interview. I believe that this process reduced my bias as the study proceeded. It was fortunate that the two focus groups were the last interviews and that most of the data came from them. The patterns revealed by the findings support the idea that the participant responses were not overly influenced by researcher bias.

Data Sources

Since the goal of the study was to probe student perceptions, data collection took the form of interviews. Observation was not used, as there was no particular behavior anticipated. Two individual interviews were followed by two group interviews (focus groups). It was expected that the most relevant data would be obtained from the focus groups, as students would react to each other's perceptions. The two individual interviews yielded data, but also served the role of refining the interview questions for the focus groups.

The first step in the interview process was to write an Interviewer's Guide and a Moderator's Guide for the individual and group interviews. These guides framed a series of questions that were to be answered during each interview and focus group. The results from each interview and focus group session were used to further refine the guides for the subsequent interviews.

The individual interviews followed a "… general interview guide approach [which] involves outlining a set of issues that are to be explored with each respondent before interviewing begins. The issues in the outline need not be taken in any particular order and the actual wording of the questions to elicit responses about those issues is not determined in advance" (Patton, 1990, pg. 280).

The goal of the interviews was to determine how the physics course affected the interviewee's perceptions about both mathematics and their achievement in mathematics. As a result, the interview questions first probed the student's middle school perceptions about math and science and then repeated those questions for ninth grade. This then led into a specific discussion of how they felt that the physics course might have affected in their year-to year perceptual change regarding mathematics. A sampling of the fifteen questions in the guide include:

- In Middle School, would someone have described you as being more of a math/science person or as more of an English/history or music/art type of person?
- In Middle School, did you think math is an important subject? If so, why?
- Have your feelings about math changed since last year?
- Do you think of yourself as being more or less of a math/science person now than you did last year?
- Did your physics class help you understand math? If so, why? Any specific examples?
- Did your physics class change your sense about the importance of math or science?
- Do you now think that math is an important subject? If so, why?

The duration of each of the individual interviews was about twenty minutes while the duration of each of the focus groups was about sixty minutes and included four students. As the topic was well defined, this proved an adequate time. Both the group and individual interviews were scheduled to fit into the normal school day so that they could be held on campus with minimum disruption of the students' schedules.

During the focus groups a pattern began to emerge. Disagreements between students developed that were not so much about whether physics was a benefit to their mathematics, but how and why it was a benefit. This was not one of my interview questions; it arose naturally through the discussions between the students in the groups. While conducting the groups, and even more so in transcribing them, a certain consistency could be seen. The position taken on this question seemed to depend on the strength of the student in mathematics, which was made clear by their answers to interview questions that had understanding that as a goal.

This led me to write a research memo suggesting the gathering of quantitative data about the student's prior and current achievement in mathematics. I followed up on that with the school and was able to obtain each student's mathematics admissions test results and their current mathematics grades.

Data Analysis

The purpose of this study was to determine how taking a mathematically intensive ninth-grade physics affected student perceptions and achievement in mathematics. The raw data primarily consisted of digital recordings of group or individual interviews with ten students.   That data was first uploaded to my computer and then fully transcribed.   The N6 program was used to manage the coding process.

The first pass at coding was descriptive only and included coding all statements regardless of whether they were likely to prove relevant. For instance, initial codes captured information as to what science and math courses the student had taken in middle school, what courses they were taking now and answers to each of the interview questions. Emergent topics were coded without regard to their relevance to the research topic. These included references that were made to the textbook, jokes that students told about physics, career or college plans, self-esteem issues, effects on their view of the world, etc.

As coding progressed patterns began to emerge that led to a few codes taking on greater importance. Other codes, while interesting for future study, did not address the research question and they were not explored further. One of the codes that took on great meaning was for statements made about the way studying physics affected the each student's view of mathematics. By browsing through all the statements with this code it became clear that there were three sub codes that should be placed under that broader code. The code *Physics Effect on Math View* (*Benefit*) was made into a "tree code" and its child codes became *Practice*, *Usefulness* and *Meaning*.

Practice referred to statements indicating that the extra time spent solving algebra problems in physics class helped the student in mathematics. Rather than doing mathematics for just one period a day, the parallel physics course nearly doubles the number of mathematics problems solved each day while offering a greater variety of problems.

Usefulness referred to statements indicating the student found the physics course made them feel that there was a greater utility to mathematics than they had realized. In some cases, this was due to the need to use mathematics in the physics course, while in other cases it referred to real world problems in physics that revealed that mathematics would be useful in later life.

Meaning referred to any statements that indicated that the subject felt that their understanding of mathematics was aided by solving problems in physics class that had a real context. For instance, using algebra to solve for the velocity of a car rather for a meaningless variable like "x".

While there was some overlap between these codes, it was generally possible to distinguish them from one another and each represented a different perspective on the benefit obtained by studying physics. In cases where two codes were comparably applicable, both codes were used.

Two other important codes were *Middle School Self Description* and *Middle School View of Math*. From the statements with these codes it was possible to get a sense of the self-perceived mathematical strength of each student prior to ninth grade. There were too few statements, typically only one or two, to do quantitative analysis of this data. However, it was possible to discern a general pattern connecting these codes to the *Practice*, *Usefulness* and *Meaning* codes.

Other very practical codes included each subject's name, so that it was possible to create intersections between any of the above codes by person. The idea of this was to identify individual patterns between the codes described above. Much of the analysis then became the process of identifying those patterns that were consistent across the group and those that varied by individual.

A trend emerged wherein students with the highest achievement in mathematics were repeatedly making statements that coded under *Practice*. Students who described themselves as having the least interest or ability in mathematics most often made statements that coded under *Meaning*. Students in the middle of the range made more statements that coded under *Usefulness*.

There were enough quotes by each student, an average of seven comments on this topic per student, to do quantitative analysis on the percentage of each explanation used by each student. I used a dummy variable approach, using a value of 100 for each statement supporting the importance of Practice, 50 for each statement supporting the importance of Usefulness and 0 for each statement supporting the importance of Meaning, to create a numerical index for each student. This gave a score, between 0 and 100, indicating where each student's perspective fell on the spectrum from Practice to Usefulness to Meaning.

I also gathered quantitative data on each subject's current level of mathematical achievement. While more complex quantitative measures were investigated, I chose the simplest approach in order to minimize any bias that might be introduced by a complex weighting scheme. I calculated the average of the first semester mathematics grades for each student using a weighted GPA approach. This approach weights course grades based on their level of difficulty, as is often done to compute a high school GPA. Geometry was weighted about 12 % higher than algebra and pre-calculus was weighted about 12% higher than geometry.

The quantitative data was first put into tabular form with Excel and then analyzed with SPSS 13.0. A correlation coefficient was generated between the dummy variable for comments and the weighted math GPA of each student. Also, a scatter plot with a best-fit trend line was generated.

Focusing on that variance made it possible to increase the validity of the study, a case of “the exception that proves the rule”. If individual variations were consistent with reasonable expectations based on all three sets of data, it would be less likely that extraneous factors, including researcher bias, were driving the subject’s statements. This would prove a useful form of triangulation.

Findings

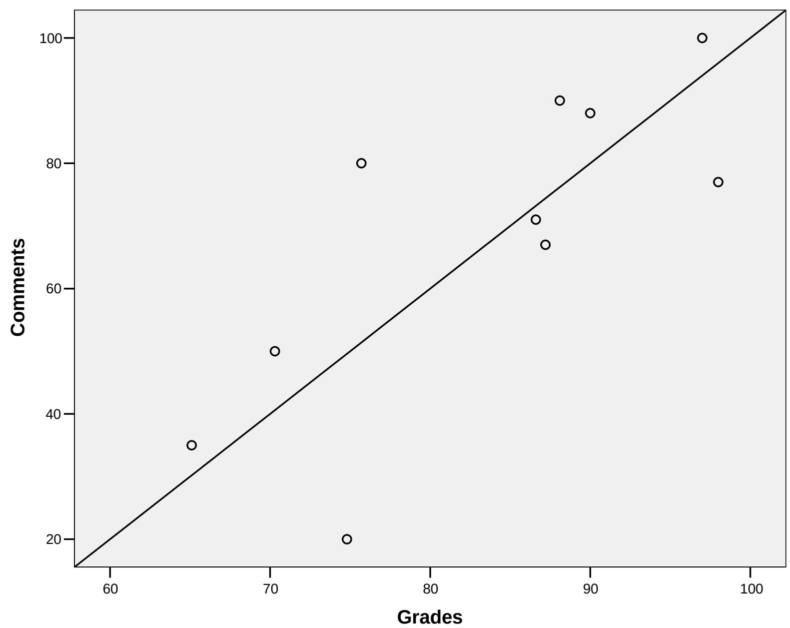
A pattern emerged as I listened to the students speak about their math backgrounds, their descriptions of themselves as students and their perceptions about the effect of physics on their performance in mathematics. Those that viewed themselves as strong in mathematics took positions very different from those that did not, while a middle group offered a third perspective.

The below table illustrates the relationship between the type of comments offered and each student’s current math grades. The “Comments” column ranges from zero, for comments invoking *Meaning*, to 50, for statements invoking *Usefulness,* to 100 for statements attributing a benefit to *Practice*. The “Grades” column contains the weighted first semester mathematics grades for each student.

Name	**Comments**	**Math Grades**
Ted	77	98
Maggie	100	97
Stan	88	90
Bob	90	88
Priyanka	67	87
Christie	71	87
Steve	80	76
Ned	20	75
Tierra	50	70
Len	35	65

Students who are strong at math primarily viewed the physics course benefiting them by giving them the opportunity to *Practice* their math skills. Those who are weak at math, viewed physics as giving *Meaning* to a subject, mathematics, that lacked meaning for them prior to that. The in-between group did adequately in math but doubted its *Usefulness* prior to studying physics.

The relationship between the students' comments and their current math grades can be seen in the scatter plot graph and the correlation table shown below. The correlation coefficient between those variable is 0.765, with a significance of less than .01.



Correlations

		Comments	Grades
Comments	Pearson Correlation	1	.765**
	Sig. (2-tailed)		.010
	N	10	10
Grades	Pearson Correlation	.765**	1
	Sig. (2-tailed)	.010	
	N	10	10

**. Correlation is significant at the 0.01 level (2-tailed).

A brief description of the results for each student is given below. I have divided the students into Strong, Mid-Level and Weak for the purpose of making the pattern clearer. However, this represents a continuum so it should be understood that the dividing line between these categories is arbitrary.

Strong Math Students

The three students who have math grades above 90% were Maggie, Stan and Ted. Maggie consistently described herself as enjoying math. She was never sure what it would be used for, but always enjoyed doing it and didn't seem concerned with its usefulness. She was able to skip ninth-grade algebra and currently has excellent math grades. All four of her comments related to the chance physics gave her to refresh her algebra skills. "It's [Physics is] hard in the beginning but once you get used to it, it has the same patterns [as algebra]."

Stan, described himself as being oriented towards math, science and technology. He skipped ninth-grade algebra and has had strong grades in math since middle school. Nine of Stan's ten comments on this topic coded under *Practice*. "I just figure that physics does contribute [to mathematics] in the sense of practicing the same algebra over and over and over again."

About two-thirds of Ted's comments coded as *Practice*, one-third as *Usefulness* and only one of his thirteen comments coded as *Meaning*. "I think physics helps you practice what you learn in algebra. What you already learned in algebra." "Physics does bring meaning to algebra. Cause now you know you learn algebra for a reason so you can use it in physics which is a real life situation." "I learned stuff in algebra and I thought I'd never use it again but now I see in physics you can really use everything you've learned."

Mid-level Math Students

Three students have math grades between 80% and 90%. These students made comments that were generally a blend of Practice and Usefulness. Eight of Bob's ten comments related to *Practice* while two of them coded to *Usefulness*. In stating that, "Fifty percent of physics is based on algebra. And you have to use algebra to get a specific answer", he was indicating both that physics gives you a lot of opportunities to practice algebra and also that algebra is useful in that you can't solve physics problems without it. He spoke again of *Usefulness* when he said that, "…math was important because most anything that you do…you need math and physics."

Seven of Christie's comments coded under *Practice*, six coded under *Usefulness* and one coded under *Meaning*. She used the analogy of driving to describe how doing algebra and physics every day, *Practice*, makes doing math more effortless. "It's like things that you do every day, you don't think about doing them. Like driving…you don't really think about. Like you do it, but you don't think about doing it." But she also talked about how taking physics made math seem more *Useful* when she said, "It just makes it easier because you're thinking OK, so this is what I could use it for cause in math its like what am I going to use this for? When am I going to use this? And when you're doing physics you're actually using it for something."

Two of Priyanka's three comments on this topic coded to *Usefulness*. In one case, she saw that in what she couldn't do in physics because she had not yet studied enough Geometry. "There's a lot of stuff we can't do in physics because we haven't learned everything in Geometry yet."

Weak Math Students

The remaining four students had weighted math averages of between 65% and 76%. Steve didn't really take strong positions on this issue. He made a total of four relevant comments that coded equally for *Usefulness* and *Practice*. His statement that "…once I came here and it's algebra and physics it helped out a lot because I had forgotten a lot of algebra" was coded under *Practice*. The other two codes were under *Usefulness*. "But now I see it [mathematics] as a little bit more important…that some of it may come in use later in life."

Ned typifies the benefits that weaker students perceive that they gain from physics. Four of his five comments were about the Meaning imparted to mathematics by physics. "I honestly think that's what saved my algebra grade. I honestly think that applying it [algebra] in physics to real-life situations is what saved my algebra grade." "I think physics really helped me with algebra because it's more of a real world application of math than anything else I had so far."

Tierra made an equal number of comments coded as *Meaning* and *Practice*, two of each. She also had one comment that was coded as *Usefulness*. In the following, she reflects on the Meaning imparted to a mathematics problem in a physics context. "When you get a solution in math class it might be just like seven. But in physics class its like…you take two hours." While the *Usefulness* brought to mathematics by physics can be seen in the following. "When we learned about equations…like changing them around…I was like all right I'm not going to need this but when I went to physics class and [saw] that same exact thing. And so I finally did it. And I was like now I need this. And all the AP students were telling me in physics you're going to be doing a lot of that so I made sure that I learned it properly."

Len has the lowest weighted math average of the group and his seven comments were almost evenly spread between *Practice*, *Usefulness* and *Meaning*. "Physics helps you understand algebra." "It's like if you use variables, how like having the variables actually meaning something. It gives algebra more meaning. Physics give algebra more meaning."

Why the difference in perspective?

The relationship between each student's perception of the benefit of studying physics and his or her initial ability and achievement in mathematics is reasonable. A student who thinks of himself or herself as mathematically inclined, and has achieved good results in mathematics, is unlikely to be wondering about the *Usefulness* and *Meaning* of mathematics as much as a student who has been struggling. In fact, a successful mathematics student would look upon physics as a new opportunity to have fun with a set of skills that they've already developed. By pushing their limits in that new field, they will see themselves growing mathematically by virtue of being able to apply their skills to a new set of problems. They would view this as benefiting them by giving them more *Practice*.

A weak mathematics student may not have yet made the transition to understanding the meaning of mathematical symbols. A big step in algebra is understanding how to translate problems from words into symbols (Kieran, 1992). This gives mathematics a context and makes it meaningful. If a student has not taken that step, mathematics can seem a particularly meaningless activity, simply moving symbols around by virtue of arbitrary meaningless rules. Every physics problem is essentially a word problem. They all begin with a description of reality and then transform that into algebraic sentences and finally to a solution. Students, who missed that in algebra, get their second chance at it in physics. Students who already made that transition, strong math students, would not see that as a benefit of physics.

The mid-level students may have learned how to do mathematics problems by virtue of hard work and practice, but their grasp of the meaning and usefulness of mathematics may be tenuous. Mathematics may not come easily to them, so they may often wonder, “why bother”. The added benefit of being able to solve physics problems that cannot be solved by those without their math skills provides an immediate benefit. To the extent that that reveals that there may be a world of such problems waiting for them in the future, answers their question even more completely.

Discussion

The purpose of this study was to determine the benefits, if any, of taking a mathematically intensive physics course in ninth grade with respect to improved student understanding of mathematics. One key question related to transfer, as there is no practical manner in which one class could result in improved learning in another unless transfer occurs between them. The literature indicated that transfer could be quite strong between mathematics and physics. This was shown to be true in this case. All the students made references to learning material in one class that they used in the other.

Additionally, the literature expressed three possible pathways for this transfer to occur. In this paper, I refer to these pathways as *Practice*, *Meaning* and *Usefulness*. This study found that all three pathways were used. However, they were used to greatly differing extents by students with different levels of achievement in mathematics. The clear pattern that was found confirms that all of these pathways are viable. Which one is used depends on the mathematical strength of the student.

The clear and reasonable pattern of the students’ comments only reinforces the fact that these were their true feelings on the subject. This is a classic case of the real meaning of the expression, “The exception that proves the rule”. The strong students who insisted that they did not gain Meaning from physics bolstered the validity of the opposing claim by weaker math students who insisted that that was the benefit of studying physics. The lack of a claim by the weaker math students, that Practice was a benefit of studying physics, only served to support that claim by the stronger math students. The importance of Usefulness to the mid-level math students mirrors that same symmetry. All of these students, with their varying levels of math achievement, were not simply saying what they wanted their peers or me to hear. They were saying what they believed.

The fact that students at very different levels of mathematics achievement all saw benefits, albeit different ones, also reveals an important opportunity. The students in this study all share one or the other of two physics classes. They all value their physics class and feel it helps them in mathematics, but for very different reasons. This may well represent an opportunity where tracking can be minimized. If a single class can help weak, mid-level and strong math students all progress in their understanding of both physics and mathematics that represents an opportunity to go beyond the need for tracking.

The physics classes at this school take a social constructivist approach, encouraging interaction between the students so that the group can progress beyond what could be expected of an individual. In this environment, students with different prior levels of mathematical achievement can teach and learn from each other. The results of this study would support the notion that groups made up of students with diverse mathematical backgrounds can all benefit from this environment.

Further research needs to be undertaken to see if the benefits discussed in this study lead to quantitative improvements in mathematics achievement. Comparing the improvements, and types of improvements, in ninth grade mathematics achievement by students in this physics course, versus students who study biology or conceptual physics in ninth grade, would be an important next step. Also, follow-up quantitative and qualitative research should be done to see what, if any, benefit is observed beyond ninth grade.

Finally, qualitative and quantitative research should be done to determine what benefits, if any, are obtained in science achievement due to the reordering of the science sequence and the institution of this physics course. The purpose of studying mathematically intensive physics in ninth grade is to improve performance in both mathematics and science, two of the highest priority goals in US education. If this new approach has merit, it is important to determine that as soon as possible so that it can be used to help achieve those two high priority national goals.

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